TRINITY COLLEGE

Year 11 Specialist Mathematics Unit 1,2
Test 3 2021

Section 1 Calculator Free
Geometry, Proof, Trigonometry

STUDENT'S NAME ______________________________

DATE: Friday 14 May **TIME:** 35 minutes **MARKS**: 35

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Consider the true statement: *If a polygon is a quadrilateral then it has four sides.*

(a) Write down the converse of this statement and state whether it is true or false, and if it is false provide a counter example. [2]

IF IT HAS FOUR SIDES THEN IT IS A QUADRILATERAL

TRUE

(b) Write down the contrapositive of this statement and state whether it is true or false, and if it is false provide a counter example. [2]

IF IT DOES NOT HAVE FOUR SIDES THEN IT IS NOT A QUADRILATERAL

TRUE

(c) Write down the inverse of this statement and state whether it is true or false, and if it is false provide a counter example. [2]

IF IT IS NOT A QUADRILATERAL THEN IT DOES NOT HAVE FOUR SIDES

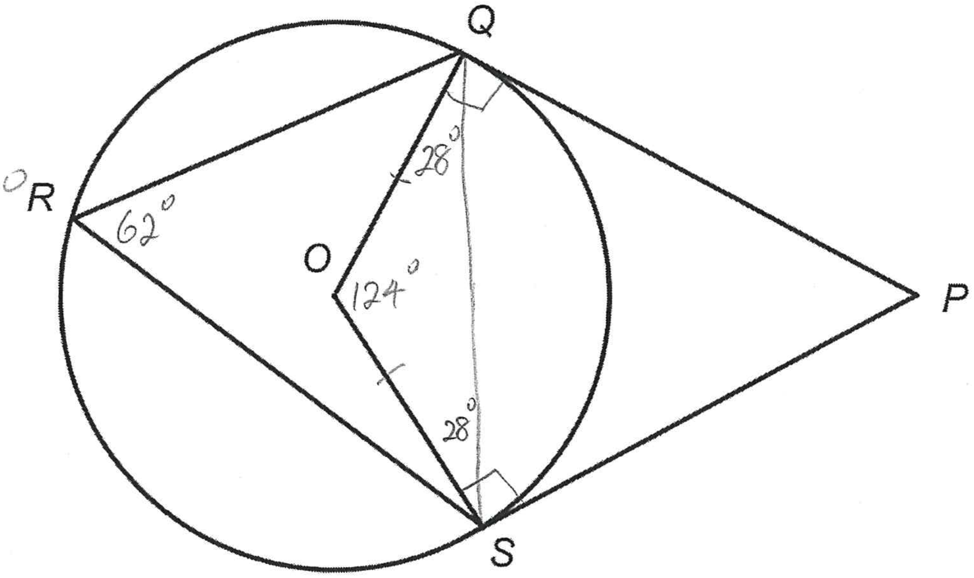
TRUE

2. (6 marks)

(a) In the diagram below, points Q, R and S lie on a circle with centre O, with tangents from P touching the circle at Q and S. If $\angle OSQ = 28°$, determine the size of $\angle QRS$ and $\angle QPS$. [3]

$\angle QRS = 62°$

$\angle QPS = 360 - 124 - 90 - 90$
$= 56°$

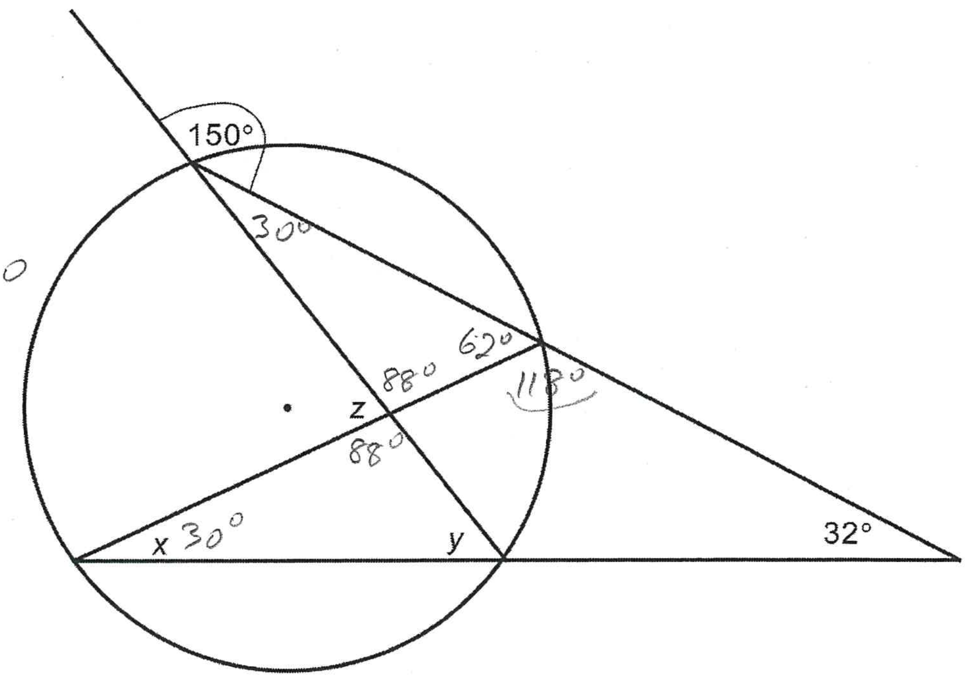


(b) In the diagram below, determine the size of the angles marked x, y, z. [3]

$x = 30°$

$z = 180 - 88$
$= 92°$

$y = 180 - 88 - 30$
$= 62°$



3. (4 marks)

Use the method of proof by contradiction to prove $\sqrt{6}$ is irrational.

TRY TO PROVE $\sqrt{6}$ IS RATIONAL

ie $\sqrt{6}$ CAN BE EXPRESSED AS A FRACTION $\frac{a}{b}$

WHERE a AND b ARE IN SIMPLEST FORM

$$\sqrt{6} = \frac{a}{b}$$

$$6 = \frac{a^2}{b^2}$$

$$6b^2 = a^2$$

$\Rightarrow a^2$ IS A MULTIPLE OF 6

$\Rightarrow a$ IS A MULTIPLE OF 6

ie $a = 6k$ FOR INTEGER k

$$6b^2 = (6k)^2$$

$$6b^2 = 36k^2$$

$$b^2 = 6k^2$$

$\Rightarrow b^2$ IS A MULTIPLE OF 6

$\Rightarrow b$ IS A MULTIPLE OF 6

SINCE BOTH a AND b ARE MULTIPLES OF 6

THEN a AND b ARE NOT IN SIMPLEST FORM

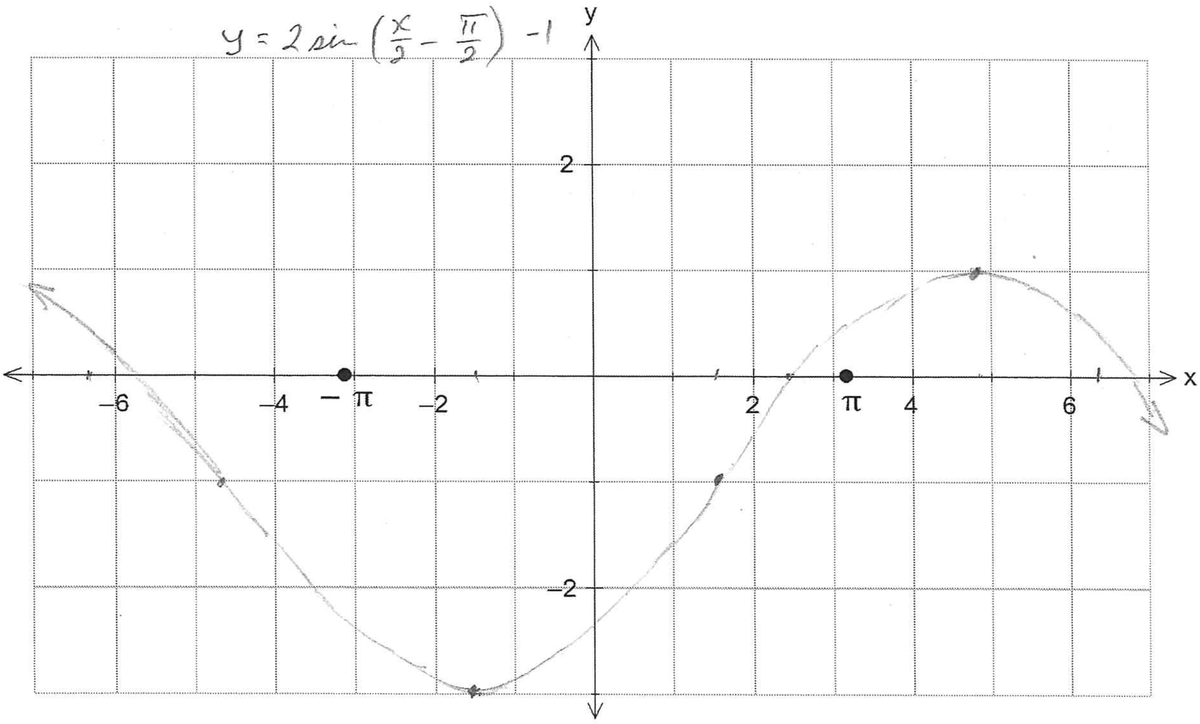
THIS IS A CONTRADICTION

$\therefore \sqrt{6}$ IS IRRATIONAL

4. (4 marks)

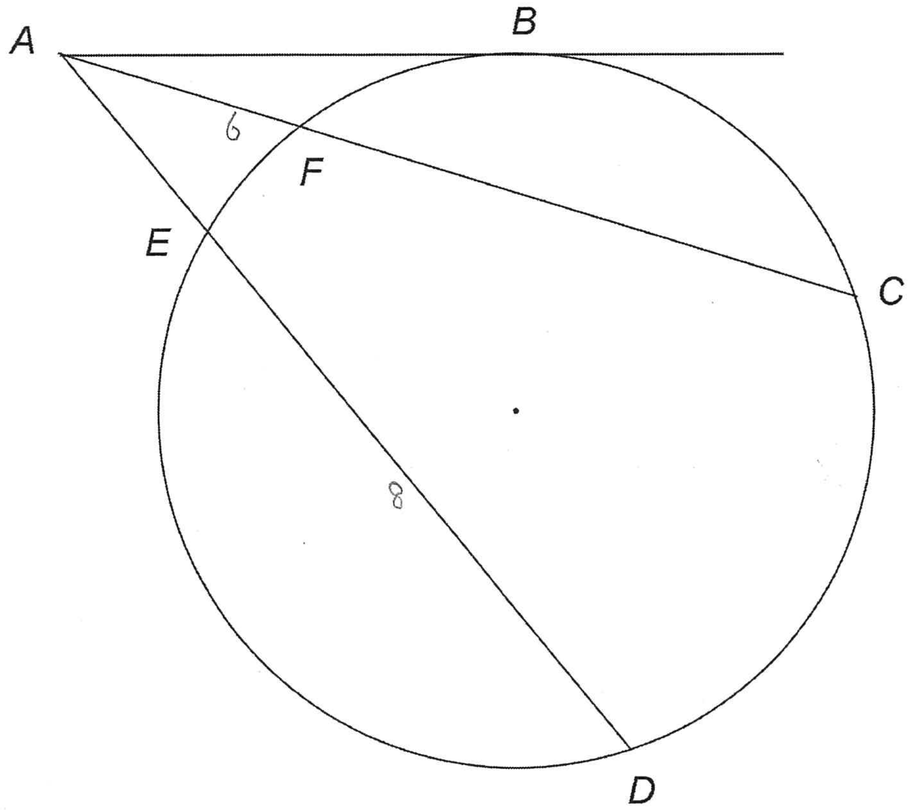
Sketch the function $y = 2\sin(\frac{x}{2} - \frac{\pi}{4}) - 1$ on the axes below.

$y = 2\sin\left(\frac{x}{2} - \frac{\pi}{2}\right) - 1$



5. (4 marks)

In the diagram below, points B, C, D, E and F lie on a circle and AB is a tangent to the circle at point B. If $AB = 10$ cm, $AF = 6$ cm and $ED = 8$ cm, determine the exact lengths of FC and AE.



$(AB)^2 = AF \cdot AC$

$100 = 6 \times AC$

$\frac{100}{6} = AC$

$\therefore FC = \frac{100}{6} - 6$

$= \frac{64}{6}$

$(AB)^2 = AE \times AD$

$100 = x \times (x+8)$

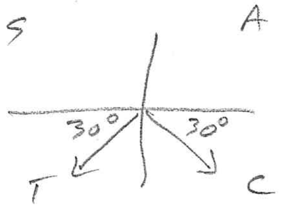
(NO CALCULATOR)

$x = AE = 6.77, \cancel{-14.77}$

6. (11 marks)

Solve the following equations.

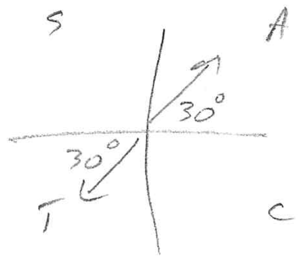
(a) $\sin\vartheta = -0.5$ $\quad -180° \le \vartheta \le 360°$ [3]



REF ANGLE $= 30^{\circ}$

$\theta = -30^{\circ}, -150^{\circ}, 210^{\circ}, 330^{\circ}$

(b) $\tan 2\vartheta = \dfrac{1}{\sqrt{3}}$ [4]



REF ANGLE $= 30^{\circ}$

$2\theta = 30^{\circ}, 210^{\circ}$

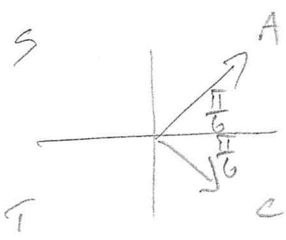
$\theta = 15^{\circ}, 105^{\circ}$

$\theta = 15^{\circ} + 90n$

$n \in \mathbb{Z}$

OR $\dfrac{\pi}{12} + \dfrac{\pi n}{2}$

(c) $2\cos(\vartheta + \dfrac{\pi}{6}) = \sqrt{3}$ $\quad 0 \le \vartheta \le 2\pi$ [4]



$\cos(\theta + \frac{\pi}{6}) = \dfrac{\sqrt{3}}{2}$

REF ANGLE $= \dfrac{\pi}{6}$

$\theta + \dfrac{\pi}{6} = \dfrac{\pi}{6}$ $\qquad \theta + \dfrac{\pi}{6} = \dfrac{11\pi}{6}$

$\theta = 0, 2\pi$ $\qquad \theta = \dfrac{10\pi}{6}$

TRINITY COLLEGE

Year 11 Specialist Mathematics Unit 1,2
Test 3 2021

Section 1 Calculator Assumed
Geometry, Proof, Trigonometry

STUDENT'S NAME ______________________________

DATE: Friday 14 May **TIME:** 15 minutes **MARKS**: 18

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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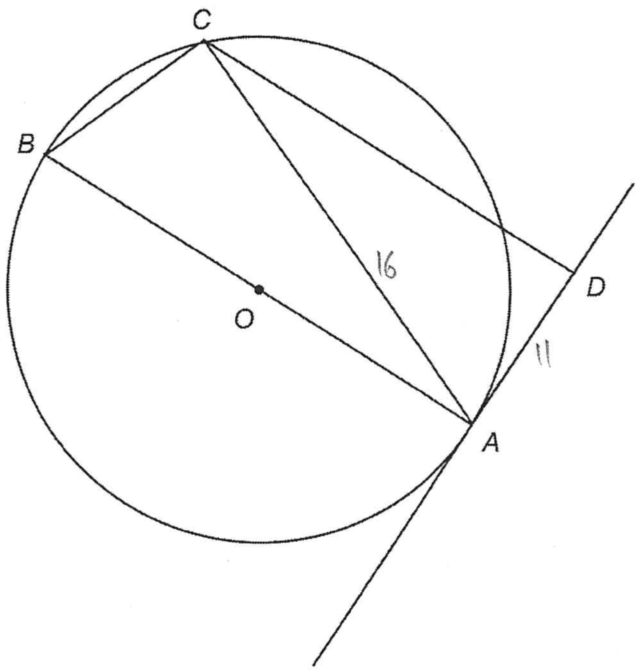
7. (7 marks)

In the diagram, AOB is the diameter of the circle. AC is a chord of the circle and CD is perpendicular to the tangent AD.



(a) Prove ΔABC is similar to ΔCAD. [3]

$\angle ACB = \angle ADC$ RT ANGLES

$\angle ABC = \angle DAC$ ALT SEGMENT

OR $\angle BAC = \angle ACD$ ALT ANGLES

$\therefore \Delta ABC \sim \Delta CAD$ AA

(b) Hence show $(AC)^2 = (AB)\times(CD)$ [2]

CORRESPONDING SIDES

$\frac{ABC}{CAD}$ $\quad \frac{AC}{CD} = \frac{AB}{AC}$

$(AC)^2 = AB \cdot CD$

(c) Determine the radius of the circle when $AC = 16$ cm and $AD = 11$ cm. [2]

$(CD)^2 = 16^2 - 11^2$

$CD = \sqrt{135}$

$(AC)^2 = AB \cdot CD$

$16^2 = AB\sqrt{135}$

$22 = AB$

$11 = r$

8. (8 marks)

The height of the tide above the mean sea level at a certain point has been modelled by the function $h(t) = 5.6\sin\dfrac{\pi t}{6}$ metres where t is the number of hours after midnight on a particular day.

Consider the graph of this function for $0 \le t \le 24$.

(a) At what time is the first high tide? [2]

$t = 3$

$\frac{\pi t}{6} = \frac{\pi}{2}$

$t = 3$

(b) How much does the tide drop from high tide to low tide? [1]

$2 \times 5.6 = 11.2$

(c) What was the height of the tide at 8.00 pm? [2]

$t = 20$

$h(20) = 5.6 \sin\left(\frac{20\pi}{6}\right)$

$= -4.8m$

4.8 m BELOW MEAN POSITION

(d) A ship needs at least 4.9 m of water above the low tide mark to safely enter the harbour in this scenario. Over the 24 hour period, state the times when the ship can safely enter the harbour. [3]

$5.6 \sin\left(\frac{\pi t}{6}\right) = -0.7$

$0 \le t \le 6.2$ HRS $11.8 \le t \le 18.2$ HRS $23.8 \le t \le 24$ HRS

9. (3 marks)

If a room contains 75 adults, use the pigeonhole principle to explain why there must be at least 11 people who have their birthday on the same day of the week.

WORST CASE SENARIO WHERE 10 PEOPLE ON EACH OF THE 7 DAYS OF THE WEEK - 70 PEOPLE.

ANY OF THE REMAINING 5 WILL INCREASE ONE DAY TO 11 PEOPLE.